PERIOD-VI

TOPIC

22 Differentiation and Integration

22.1. DIFFERENCE QUOTIENT FORMULA

In single-variable calculus, the difference quotient is usually the name for the expression, which taken to the limit when h approaches 0, gives the derivative of the function f. Difference Quotient Formula is used to find the slope of the line that passes through two points. It is also used in the definition of the derivative.

$$\text{Difference Quotient Formula} = \frac{f(x+h) - f(x)}{h}$$

Example 1. *Find the difference of the function $f(x) = 3x - 5$.*

Solution. Using the difference quotient formula

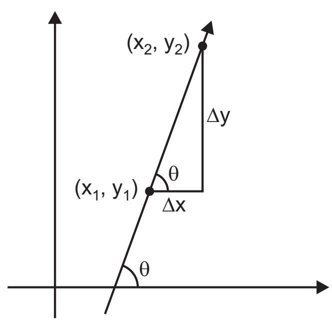
$$\begin{aligned} \text{Difference quotient of } f(x) &= [f(x + h) - f(x)]/h \\ &= [3(x + h) - 5) - (3x - 5)]/h \\ &= [3x + 3h - 5 - 3x + 5]/h = [3h]/h = 3 \end{aligned}$$

22.2. WHAT IS SLOPE?

The slope of a line is defined as the change in y-coordinate with respect to the change in x-coordinate of that line. The net change in y-coordinate is Δy, while the net change in the x-coordinate is Δx. So, the change in y-coordinate with respect to the change in x-coordinate can be written as,

$$m = \Delta y/\Delta x$$

where m is the slope



Note that $\tan \theta = \Delta y/\Delta x$

We also refer this $\tan \theta$ to be the slope of the line.

22.3. SLOPE OF A LINE

The slope of the line is the ratio of the rise to the run, or rise divided by the run. It describes the steepness of line in the coordinate plane. Calculating the slope of a line is similar to finding the slope between two different points. In general, to find the slope of a line, we need to have the values of any two different coordinates on the line.

Example 2. *Given a line with the equation $2y = 8x + 9$, find its slope.*

Solution. We know that the general formula of the slope is given as, $y = mx + b$

Hence, we try to bring the equation to this form. We make the coefficient of $y = 1$, and hence we get,

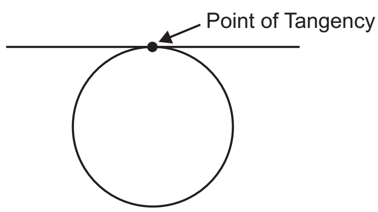
$$y = 4x + 4.5$$

Clearly, the coefficient of x is found to be 4. Hence, our slope will be same as the coefficient of x.

The slope is 4.

22.4. WHAT IS TANGENT LINE?

The **tangent line** of a curve at a given point is a line that just touches the curve (function) at that point. The tangent line in calculus may touch the curve at any other point(s) and it also may cross the graph at some other point(s) as weill. The point at which the tangent is drawn is known as the "point of tangency". We can see the tangent of circle drawn here.



22.5. CONCEPT OF LIMIT

Let $f(x)$ be a function defined for all x in the nbd of 'a' except possibly at 'a'. Then, l is said to be the limiting value of $f(x)$ as x tends to a. If the numercial difference between $f(x)$ and l can be made as small as we please by taking x sufficiently close to 'a' but not equal to 'a'.

We write this as: $\lim_{x \to a} f(x) = l$

Definition. Let f be a function defined in a nbd of a except possible at a. Then, a real number l is said to be a limit of f as tends to a if for any $\varepsilon > 0$, however small, there exists $\delta > 0$ (depending upon ε) such that:

$| f(x) - l | < \varepsilon$, whenever $0 < | x - a | < \delta$.

i.e. $l - \varepsilon < f(x) < l + \varepsilon$, whenever $x \in (a - \delta, a) \cup (a, a + \delta)$

We write $\lim_{x \to a} f(x) = l$

Remark. The limit of f at a, if exists, will continue to exist and be the same if we change the value of f at a only.

Left Hand Limit of a Function

Let $f(x)$ be a function of x.

If $f(x)$ approaches to l as $x \to a^-$, then l is called the left hand limit of the function and we write it as:

$$\lim_{x \to a^-} f(x) = l$$

Right Hand Limit of a Function

Let $f(x)$ be a function of x if $f(x)$ approaches to l as $x \to a^+$, then l is called the right hand limit of the function and we write it as:

$$\lim_{x \to a^+} f(x) = l$$

Remark.

(*i*) $\lim_{x \to a} f(x)$ exists if and only if

$\lim_{x \to a^-} f(x)$ and $\lim_{x \to a^+} f(x)$ both exist and are equal i.e.,

$\lim_{x \to a} f(x)$ exists iff $\lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x)$

(*ii*) If $x \to a^-$ and $x = a - h$, then we have $h = (a - x)$ and as $x \to a^-$, the difference $a - x$ (= h) is positive and is close to zero.

Fundamental Theorems on Limits

Here we list some of the fundamental results involving limits of functions. The proofs of these are beyond the scope of this book.

(*i*) If $f(x) = k$, a constant function, then $\lim_{x \to a} f(x) = k$

(*ii*) $\lim_{x \to a}[k\, f(x)] = k \lim_{x \to a} f(x)$ where : k is a consant

(*iii*) $\lim_{x \to a}[f(x) + g(x)] = \lim_{x \to a} f(x) + \lim_{x \to a} g(x)$

(*iv*) $\lim_{x \to a}[f(x) - g(x)] = \lim_{x \to a} f(x) - \lim_{x \to a} g(x)$

(*v*) $\lim_{x \to a}[f(x) \cdot g(x)] = \lim_{x \to a} f(x) \cdot \lim_{x \to a} g(x)$

(*vi*) $\lim_{x \to a} \dfrac{f(x)}{g(x)} = \dfrac{\lim_{x \to a} f(x)}{\lim_{x \to a} g(x)}$, provided $\lim_{x \to a} g(x) \neq 0$

(*vii*) If $f(x) \leq g(x)$ for all x, then: $\lim_{x \to a} f(x) \leq \lim_{x \to a} g(x)$

(*viii*) $\lim_{x \to a}\left(\dfrac{1}{f}\right)(x) = \lim_{x \to a} \dfrac{1}{f(x)} = \dfrac{1}{\lim_{x \to a} f(x)}$, provided $\lim_{x \to a} f(x) \neq 0$

(*ix*) $\lim_{x \to a}[f(x)]^n = \left[\lim_{x \to a} f(x)\right]^n$

Example 3. *Evaluate the following limits:*

(*i*) $\lim_{x \to 2} \dfrac{x^3 - 8}{x^2 - 4}$ (*ii*) $\lim_{x \to 2} \dfrac{x^2 - 4}{x + 2}$ (*iii*) $\lim_{x \to 2} \dfrac{x^2 - 5x + 6}{x^2 - 4}$

(*iv*) $\lim_{x \to -1} \dfrac{x^3 + 1}{x + 1}$ (*v*) $\lim_{x \to 0} \dfrac{2x^2 + 3x + 4}{x^2 + 3x + 2}$

Solution. (*i*) When $x = 2$, the given expression assumes the indeterminate form $\left(\dfrac{0}{0}\right)$.

$$\therefore \quad \lim_{x \to 2} \frac{x^3 - 8}{x^2 - 4} = \lim_{x \to 2} \frac{x^3 - 2^3}{x^2 - 2^2} \qquad \left(\text{Form } \frac{0}{0}\right)$$

$$= \lim_{x \to 2} \frac{(x-2)(x^2 + 2x + 4)}{(x-2)(x+2)}$$

$$\left[\because \quad a^3 - b^3 = (a - b)(a^2 + ab + b^2)\right]$$

$$= \lim_{x \to 2} \frac{x^2 + 2x + 4}{x + 2} = \frac{(2)^2 + 2(2) + 4}{2 + 2} = \frac{12}{4} = 3$$

(*ii*) We have, $\lim_{x\to 2}\frac{x^2-4}{x+2} = \frac{(2)^2-4}{2+2} = \frac{4-4}{4} = \frac{0}{4} = 0$

(*iii*) When $x = 2$, the given expression assumes the indeterminate form $\left(\frac{0}{0}\right)$

$$\therefore \quad \lim_{x\to 2}\frac{x^2-5x+6}{x^2-4} = \lim_{x\to 2}\frac{(x-2)(x-3)}{(x+2)(x-2)} \qquad \left(\text{Form } \frac{0}{0}\right)$$

$$= \lim_{x\to 2}\frac{x-3}{x+2} = \frac{2-3}{2+2} = -\frac{1}{4}$$

(*iv*) When $x = -1$, the given expression assumes the indeterminate form $\left(\frac{0}{0}\right)$

$$\therefore \quad \lim_{x\to -1}\left(\frac{x^3+1}{x+1}\right) = \lim_{x\to -1}\frac{x^3+1^3}{x+1} \qquad \left(\text{Form } \frac{0}{0}\right)$$

$$[\because \quad a^3 - b^3 = (a-b)(a^2+ab+b^2)]$$

$$= \lim_{x\to -1}\frac{(x+1)(x^2-x+1)}{x+1} = \lim_{x\to -1}(x^2-x+1)$$

$$= (-1)^2 - (-1) + 1 = 1 + 1 + 1 = 3$$

(*v*) We have,

$$\lim_{x\to 0}\frac{2x^2+3x+4}{x^2+3x+2} = \frac{0+0+4}{0+0+2} = \frac{4}{2} = 2.$$

22.6. DIFFERENTIATION

In calculus, differentiation is one of the two important concepts apart from integration. Differentiation is a method of finding the derivative of a function. Differentiation is a process, in Maths, where we find the instantaneous rate of change in function based on one of its variables. The most common example is the rate change of displacement with respect to time, called velocity. The opposite of finding a derivative is anti-differentiation.

If x is a variable and y is another variable, then the rate of change of x with respect to y is given by dy/dx. This is the general expression of derivaive of a function and is represented as $f'(x) = dy/dx$, where $y = f(x)$ is any function.

22.7. CONCEPT OF DIFFERENTIATION

1. **Product rule of Differentiation:**
   Definition: If u and v are two differentiable functions of x, then

$$\frac{d}{dx}(uv) = u \cdot \frac{d}{dx} + v \cdot \frac{du}{dx}$$

2. **Quotient rule of Differentiation:**
   Definition: If u and v are differentiable of x, then

Then $$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{du}{dx}}{v^2}$$

3. **Derivative of function of a function (Chain rule):**
   Definition: If $y = f(u)$ and $u = g(x)$ are differentiable function of u and x respecitvely.

Then $$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

Derivative of Algebraic Functions:

If $y = x^n$, then $\frac{d}{dx}x^n = nx^{n-1}$

Derivative of Algebraic Functions:

If $y = x$, then $\frac{d}{dx}(c) = 0$

Example 4. *Differentiate the following functions w.r.t. x.*

(*i*) $ax^3 + bx^2 + cx + d$ (*ii*) $\left(x + \frac{1}{x}\right)\left(x^2 + \frac{1}{x^2}\right)$

(*iii*) $(2x + 3)^2$ (*iv*) $\left(x + \frac{1}{x}\right)\left(\sqrt{x} + \frac{1}{\sqrt{x}}\right)$

Solution. (*i*) Let $y = ax^3 + bx^2 + cx + d$

$\therefore$ $$\frac{dy}{dx} = 3ax^2 + 2bx + c.1 + d \qquad \left[\because \frac{d}{dx}(x^n = nx^{n-1})\right]$$

$\Rightarrow \qquad \dfrac{dy}{dx} = 3ax^2 + 2bx + c$

(*ii*) $\qquad y = \left(x + \dfrac{1}{x}\right)\left(x^2 + \dfrac{1}{x^2}\right)$

$= x^3 + \dfrac{1}{x} + x + \dfrac{1}{x^3}$

$\therefore \qquad \dfrac{dy}{dx} = 3x^2 - 1x^{-2} + 1 - 3x^{-4} = 3x^2 - \dfrac{1}{x^2} + 1 - \dfrac{3}{x^4}$

$\left[\because \dfrac{d}{dx}(x^n = nx^{n-1})\right]$

(*iii*) Let $\qquad y = (2x + 3)^2$

$= 4x^2 + 9 + 2(2x)(3) \quad [\because (a + b)^2 = a^2 + b^2 + 2ab]$

$= 4x^2 + 9 + 12x$

$\therefore \qquad \dfrac{dy}{dx} = 8x + 12 + 0 = 8x + 12 \qquad \left[\because \dfrac{d}{dx}(x^n = nx^{n-1})\right]$

(*iv*) Let $\qquad y = \left(x + \dfrac{1}{x}\right)\left(\sqrt{x} + \dfrac{1}{\sqrt{x}}\right)$

$= x\sqrt{x} + \dfrac{\sqrt{x}}{x} + \dfrac{x}{\sqrt{x}} + \dfrac{1}{x\sqrt{x}}$

$= x^{3/2} + \dfrac{1}{\sqrt{x}} + \sqrt{x} + \dfrac{1}{x^{3/2}}$

$\Rightarrow \qquad y = x^{3/2} + x^{-1/2} + x^{1/2} + x^{-3/2}$

$\therefore \qquad \dfrac{dy}{dx} = \dfrac{3}{2}x^{\frac{3}{2}-1} + \left(-\dfrac{1}{2}\right)x^{-\frac{1}{2}-1} + \dfrac{1}{2}x^{\frac{1}{2}-1} + \left(-\dfrac{3}{2}\right)x^{-\frac{3}{2}-1}$

$\left[\because \dfrac{d}{dx}(x^n = nx^{n-1})\right]$

$= \dfrac{3}{2}x^{1/2} - \dfrac{1}{2}x^{-3/2} + \dfrac{1}{2}x^{-1/2} - \dfrac{3}{2}x^{-5/2}$

$= \dfrac{3}{2}\sqrt{x} - \dfrac{1}{2x^{3/2}} + \dfrac{1}{2x^{1/2}} - \dfrac{3}{2x^{5/2}}$

$$= \frac{3}{2}\sqrt{x} - \frac{1}{2x\sqrt{x}} + \frac{1}{2\sqrt{x}} - \frac{3}{2x^2\sqrt{x}}$$

$$= \frac{1}{2}\left(3\sqrt{x} - \frac{1}{x\sqrt{x}} + \frac{1}{\sqrt{x}} - \frac{3}{x^2\sqrt{x}}\right)$$

22.8. INTEGRATION

Integration is a method of adding or summing up the parts to find the whole. It is a reverse process of differentiation, where we reduce the functions into parts. This method is used to find the summation under a vast scale. Calculation of small addition problems in an easy task which we can do manually or by usiung calculations as well. But for big addition problems, where the limits could reach to even infinity. Integration methods are used. Integration and differentiation both are important parts of calculus. The concept level of these topics is very high. Hence, it is introduced to us at higher secondary classes and thein in engineering or higher education. To get an in-depth knowledge of integration, read the complete article here.

Important Extension Formulae of Standard Integral Forms

(*i*) $\int x^n \cdot dx = \frac{x^{n+1}}{n+1} + c;\ (n \neq -1)$

$\Rightarrow \int (ax+b)^n \cdot dx = \frac{(ax+b)^{n+1}}{(n+1)\cdot a} + c; (n \neq -1)$

(*ii*) $\int \frac{1}{x} \cdot dx = \log|x| + c$

$\Rightarrow \int \frac{1}{(ax+b)} \cdot dx = \frac{\log|ax+b|}{a} + c$

(*iii*) $\int a^x \cdot dx = \frac{a^x}{\log a} + c;\ (a > 0,\ a \neq 1)$

$\Rightarrow \int a^{mx+b} \cdot dx = \frac{a^{mx+b}}{m \cdot \log a} + c;\quad (a > 0;\ a \neq 1)$

(*iv*) $\int e^x \cdot dx = e^x + c$

$\Rightarrow \int e^{mx+b} \cdot dx = \frac{e^{mx+b}}{m} + c$

(*v*) $\int \cos x \cdot dx = \sin x + c$

$\Rightarrow \int \cos (ax + b)dx = \frac{\sin (ax + b)}{a} + c$

(*vi*) $\int \sin x \, dx = - \cos x + c$

$\Rightarrow \int \sin (ax + b)dx = -\frac{\cos (ax + b)}{a} + c$

(*vii*) $\int \sec^2 x \, dx = \tan x + c$

$\Rightarrow \int \sec^2 (ax + b)dx = \frac{\tan (ax + b)}{a} + c$

(*viii*) $\int \text{cosec}^2 x + dx = - \cot x + c$

$\Rightarrow \int \text{cosec}^2 (ax + b)dx = \int \frac{- \cot (ax + b)}{a} + c$

(*ix*) $\int \sec x \tan x \, dx = \sec x + c$

$\Rightarrow \int \sec (ax + b) \tan (ax + b) \, dx = \frac{\sec(ax + b)}{a} + c$

(*x*) $\int \text{cosec } x + \cot x \, dx = - \text{cosec } x + c$

$\Rightarrow \int \text{cosec } (ax + b) + \cot(ax + b)dx = \int -\frac{\text{cosec } (ax + b)}{a} + c$

Example 5. *Evaluate the following integrals*

(*i*) $\int (1 - x)\sqrt{x} \, dx$ (*ii*) $\int \sqrt[3]{x - dx}$

(*iii*) $\int a^x \cdot e^x \, dx$ (*iv*) $\int (x^2 - 2x + 4)^2 \cdot dx$

(*v*) $\int 9^{x+2} \, dx$ (*iv*) $3^{2\log_3 x} \, dx$

Solution. (*i*) $\int (1 - x)\sqrt{x}dx = \int (\sqrt{x} - x\sqrt{x})dx = \int x^{1/2} - \int x^{3/2}dx$

$\left[\because \int x^n \, dx = \frac{x^{n+1}}{n+1} + c\right]$

(*ii*) $\int \sqrt[3]{x}\, dx = \int x^{1/3}\, dx = \dfrac{x^{1/3+1}}{\frac{1}{3}+1} + c$ $\left[\because \int x^n\, dx = \dfrac{x^{n+1}}{n+1} + c\right]$

(*iii*) $\int a^x \cdot e^x\, dx = \int (ae)^x\, dx = \dfrac{(ae)^x}{\log (ae)} + c$

$\left[\because \int a^x dx = \dfrac{a^x}{\log a} + c\right]$

(*iv*) $\int (x^2 - 2x + 4)^2\, dx$

$[\because \quad (a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca]$

$= \int (x^4 + 4x^2 + 16 - 4x^3 - 16x + 8x^2) dx$

$= \int (x^4 - 4x^3 + 12x^2 - 16x + 16) dx$

$= \int x^4\, dx - 4\int x^2\, dx - 16\int x\, dx + 16\int dx$

$\left[\because \int x^n\, dx = \dfrac{x^{n+1}}{n+1} + c\right]$

$= \dfrac{x^5}{5} - \dfrac{4x^4}{4} + 12\dfrac{x^3}{3} - 16\dfrac{x^2}{2} + 16x + c$

$= \dfrac{1}{5}x^5 - x^4 + 4x^3 - 8x^2 + 16x + c$

(*v*) $\int 9^{x+2}\, dx = \int 9^x \cdot 9^2\, dx$

$= \int 81.9^x dx = 81\int 9^x dx$

$= 81\left(\dfrac{9^x}{\log 9}\right) + c$ $\left[\because \int a^x\, dx = \dfrac{a^x}{\log a} + c\right]$

(*vi*) $\int 3^{2\log_3 x}\, dx = \int 3^{\log_3 x^2}\, dx$ $[\because \quad m \log n = \log n^m]$

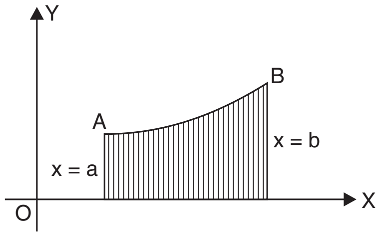
$= \int x^2\, dx = \dfrac{x^{2+1}}{2+1} + c$ $[\because \quad a^{\log_a x} = x]$

$= \dfrac{1}{3}x^3 + c$

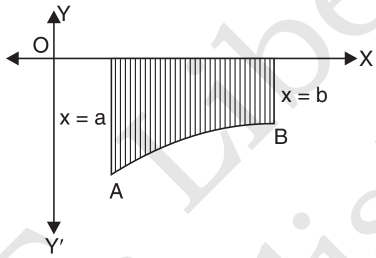
22.9. AREA UNDER THE CURVE

1. The area under the curve $y = f(x)$, above x-axis between the ordinates $x = a$ and $x = b$ is given by

$$\int_a^b y.dx = \int_a^b f(x)\,dx$$



(a)



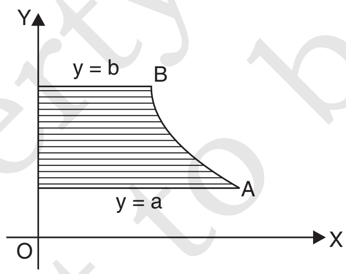
(b)

2. The area bounded by the curve $y = f(x)$, below x-axis between the ordinates $x = a$ and $x = b$ is given by

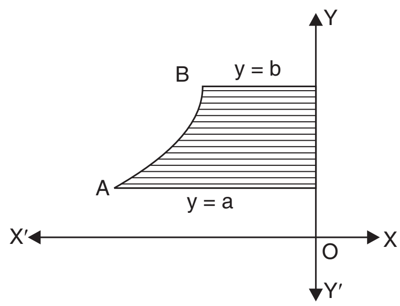
$$\int_a^b -y\,dx = -\int_a^b y\,dx = -\int_a^b f(x)\,dx$$

3. The area bounded by the curve $x = f(y)$, y-axis between the abscissae $y = a$ and $y = b$ is given by

$$\int_a^b x\,dy = \int_a^b f(y)\,dy$$



(a)



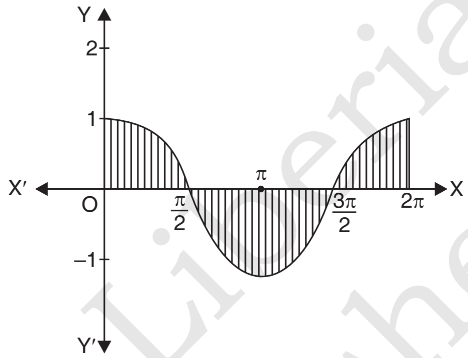
(b)

4. The area bounded by the curve $x = f(y)$, y-axis between the abscissae $y = a$ and $y = b$ is given by

$$\int_a^b -x\,dy = \int_a^b f(y)\,dy$$

Example 6. *Find the area bounded by the curve y = cos x, x-axis and the ordinates x = 0 and x = 2π.*

Solution. The equation of the given curve is $y = \cos x$



Now $\cos x > 0$ when $x \in \left(0, \frac{\pi}{2}\right)$

$\cos x < 0$ when $x \in \left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$

$\cos x > 0$ when $x \in \left(\frac{3\pi}{2}, 2\pi\right)$

$\Rightarrow$ $f(x)$ changes sign in the given interval.

Table of values of y corresponding to values of x from 0 to 2π is given by

x	0	$\pi/2$	π	$3\pi/2$	2π
y	1	0	– 1	0	1

By joining these points with a free hand we obtain a rough sketch of the curve as shown in the figure,

$\therefore$ Required area

= Area of shaded region

$$= \int_0^{2\pi} |y| \cdot dx = \int_0^{\pi/2} |y| \cdot dx + \int_{\pi/2}^{3\pi/2} |y| \cdot dx + \int_{3\pi/2}^{2\pi} |y| \cdot dx$$

$$= \int_0^{\pi/2} |\cos x\, dx + \int_{\pi/2}^{3\pi/2} -\cos x\, dx + \int_{3\pi/2}^{2\pi} \cos x\, dx$$

$$= [\sin x]_0^{\pi/2} - [\sin x]_{\pi/2}^{3\pi/2} + [\sin x]_{3\pi/2}^{2\pi}$$

$$= \left[\sin\frac{\pi}{2} - \sin 0\right] - \left[\sin\frac{3\pi}{2} - \sin\frac{\pi}{2}\right] + \left[\sin 2\pi - \sin\frac{3\pi}{2}\right]$$

$= (1 – 0) – (– 1 – 1) + [0 – (– 1)] = 1 + 2 + 1$

$= 4$ sq. units

22.10. INDEFINITE INTEGRAL

Definition. Let $f(x)$ *be a function. Then the family of all its primitives (or antiderivatives) is called the indefinite integral of f(x) and is denoted by* $\int f(x)\,dx$.

The symbol $\int f(x)\,dx$ is read as the indefinite integral of $f(x)$ with respect to x.

Thus, $\frac{d}{dx}(\phi(x)+C) = f(x) \Leftrightarrow \int f(x)\,dx = \phi(x) + C$

where $\phi(x)$ is primitive of $f(x)$ and C is an arbitrary constant known as the *constant of integration.*

Here, $\int$ is the integral sign, $f(x)$ is the integrand, x is the variable of integration and dx is the element of integration or differential of x.

22.11. INDEFINITE INTEGRALS OF TRIGONOMETRIC FUNCTIONS

Example 7. *Evaluate the following integrals:*

(*i*) $\int tan^2 x \cdot dx$ (*ii*) $\int \sqrt{1 - sin\ 2x} \cdot dx$

(*iii*) $\int (sin\ x + cos\ x) \cdot dx$ (*iv*) $\int cosec\ x\ (cosec\ x + cot\ x) \cdot dx$

(*v*) $\int \frac{1 - sin\ x}{cos^2 x} \cdot dx$

Solution. (*i*) $\int \tan^2 x\ dx = \int (\sec^2 x - 1)dx$

$[\because \sec^2 A - \tan^2 A = 1]$

$= \int \sec^2 x \cdot 1 - \int 1 \cdot dx = \tan x - x + c$

(*ii*) $\int \sqrt{1 - \sin 2x} \cdot dx = \int [(\cos^2 x + \sin^2 x) - 2 \sin x \cos x]^{1/2} \cdot dx$

$[\because \cos^2 A + \sin^2 A = 1, \sin 2A = 2 \sin A \cos A]$

$= \int [(\cos x - \sin x)^2]^{1/2} \cdot dx$

$$= \int (\cos x - \sin x) \cdot dx$$

$$= \int \cos x\, dx - \int \sin x\, dx$$

$= \sin x - (-\cos x) + c$

$= \sin x + \cos x + c$

(*iii*) $\int (\sin x + \cos x) \cdot dx = \int \sin x\, dx + \int \cos x\, dx$

$= -\cos x + \sin x + c$

(*iv*) $\int \operatorname{cosec} x (\operatorname{cosec} x + \cot x) \cdot dx$

$$= \int \operatorname{cosec}^2 x\, dx + \int \operatorname{cosec} x \cot x \cdot dx$$

$= -\cot x - \operatorname{cosec} x + c$

(*v*) $\int \frac{1 - \sin x}{\cos^2 x} dx = \int \left(\frac{1}{\cos^2 x} - \frac{\sin x}{\cos^2 x} \right) \cdot dx$

$$= \int (\sec^2 x - \sec x \tan x) \cdot dx$$

$$= \int \sec^2 x \cdot dx - \int \sec x \tan x \cdot dx$$

$= \tan x - \sec x + c$

EXERCISE

1. Find the difference quotient of function f is defined by $f(x) = 2x + 5$.

2. Evaluate the following limits:

(*i*) $\lim_{x \to 1/2} \frac{8x^3 - 1}{16x^4 - 1}$ (*ii*) $\lim_{x \to \pi/2} \left(\frac{\sin^2 x + \cos 2x}{x} \right)$

(*iii*) $\lim_{x \to 1} \left(\frac{2}{1 - x^2} + \frac{1}{x - 1} \right)$

3. Evaluate the following limits:

(*i*) $\lim_{x \to 2} \left(\frac{x}{x - 2} - \frac{4}{x^2 - 2x} \right)$ (*ii*) $\lim_{x \to 3} (x^2 - 9) \left[\frac{1}{x + 3} + \frac{1}{x - 3} \right]$

(*iii*) $\lim_{x \to 0} \frac{(1 + x)^6 - 1}{(1 + x)^2 - 1}$ (*iv*) $\lim_{x \to 3} \left(\frac{1}{x - 3} - \frac{2}{x^2 - 4x + 3} \right)$

(*v*) $\lim_{x \to -3} \frac{x^3 + 4x^2 + 4x + 3}{x^2 + 2x - 3}$

4. Differentiable the following functions w.r.t. x.

(*i*) $(x^2 + 4x + 5)$

(*ii*) $(2x + 1)^{1/3} (x + 1)$

(*iii*) $(x^2 + 1) (x^2 + x + 4)$

(*iv*) $(3x + 8)^{7/3} + (x + 7)^{-3}$

5. Differentiable the following functions w.r.t. x.

(*i*) $\dfrac{2x+3}{x^2-5}$

(*ii*) $\dfrac{3x+2}{(x+5)(2x+1)+3}$

(*iii*) $\sqrt{\dfrac{1+x}{1-x}}$

(*iv*) $\dfrac{x^4+1}{x^2+1}$

6. Evaluate the following integrals:

(*i*) $\int \sqrt{ax+b}\, dx$

(*ii*) $\int (4x^3 - 4x^{-5})dx$

(*iii*) $\int \dfrac{x^3 - x^2 + x - 1}{x-1} dx$

(*iv*) $\int \dfrac{x^2}{1+x^2} dx$

(*v*) $\int (x-3)^2 \cdot \sqrt{x}\, dx$

7. Find the area of the curve $y = x^2 - 2x$ lying below the x-axis.